Grade Level/Course:
Algebra 1

Lesson/Unit Plan Name:
Shifting Linear Equations in Function Notation.

Rationale/Lesson Abstract:
This lesson is designed to build on students' knowledge of relating graphs to the parent linear function $y = x$. Student will take this understanding to apply it to relating the graphs of one function to another, such that they are able to identify how the graph of a linear function is affected by replacing $f(x)$ with $f(x)+k$ and $f(x+k)$.

Timeframe:
2- 50 minute class periods
Day 1: Warm Up, section #1, and section #2 (15-20 minutes each part)
Day 2: Section #3, section #4, and sort activity (15-20 minutes each part)
Extension/Closure: End of day 2 if time permits, homework, or can be given on another day

Common Core Standards

F-BF.B.3: Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative).

F-IF.C.7.A: Graph linear and quadratic functions and show intercepts, maxima, and minima.

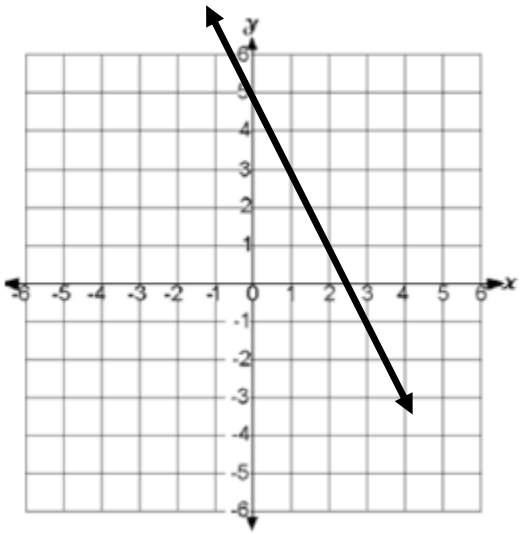
Instructional Resources/Materials:
Warm up, Student Note-Taking Guide, and Shifting Linear Equations in Function Notation Sort.

In preparation:

- Copy the Note Taking Guide for all students
- Copy Shifting Linear Equations in Function Notation Sort for each pair of students. Cut out the columns of the graphs and leave intact. Cut out all of the $f(x)$ and $g(x)$ pieces. Place pieces in a large envelope or binder clip the pieces to the intact columns out of order.

Warm Up: Attached
Start by discussing the warm up questions. It is crucial to this lesson that students understand how to graph a line in slope-intercept form, have strong sense of using function notation, and understand that any line is related to the parent function $y = x$.

Answers to Warm Up



Slope is -2 and the *y*-intercept is 5

a. ***y* = *x* + 4 is equivalent to *y*=*x* shifted up 4 units**

b. Answers may vary
examples:

$y = x + \underline{\mathbf{2}}$ is equivalent to $y = x$ **shifted up 2 units**

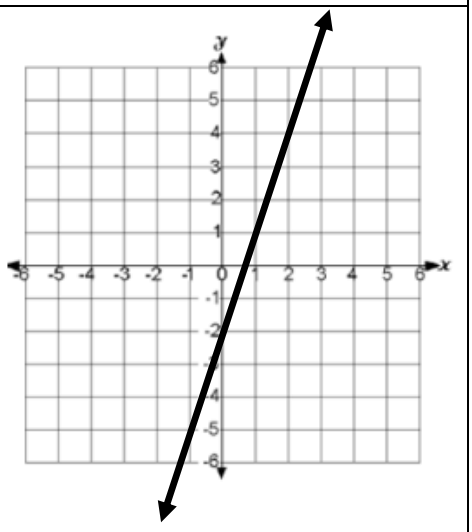
$y = x - \underline{\mathbf{5}}$ is equivalent to $y = x$ **shifted down 5 units**

1. $f(x+2)$ ⟶ d. $4x + 6$
   $\boldsymbol{f(x + 2) = 4(x + 2) – 2}$
   $\boldsymbol{= 4x + 8 - 2}$
   $\boldsymbol{= 4x + 6}$
2. $f(x) – 2$ ⟶ a. $(4x - 2) - 2$
   $\boldsymbol{f(x) - 2 = (4x - 2) – 2}$

3. $f(x) + 2$ ⟶ c. $4x$
   $\boldsymbol{f(x) + 2 = (4x – 2) + 2}$
   $\boldsymbol{= 4x}$

4. $f(x – 2)$ ⟶ b. $4(x – 2) - 2$
   $\boldsymbol{f(x – 2) = 4(x – 2) – 2}$

x	*y*
-2	-8
-1	-5
0	-2
1	1
2	4



Activity/Lesson:
Next, walk through filling out the Shifting Linear Equations in Function Notation Note Taking Guide.

For question #1: Compare the graphs of $f(x)$ and $f(x) + k$.

- a. With the students, graph $f(x) = x$ and $g(x) = x + 3$. The students will graph on their worksheet.
- b and c. Have the students turn to a partner and discuss the similarities and differences of $f(x)$ and $g(x)$. They should also discuss how the functions relate visually. Have students share, with the class, what their partner thought about the visual relationship. Make sure that there is a focus on the vertical relationship when students are sharing.
- d. With the students, set up and solve the equation $g(x) = f(x) + k$ for the value of k and then write $g(x)$ in terms of f(x). Make sure students understand that we are simply substituting x for $f(x)$ and $x + 3$ for $g(x)$, then solving k.
- e. With the students, graph $h(x)$. Tell the students to turn to their partner and discuss how $h(x)$ and $f(x)$ relate visually. Have students share, with the class, what their partner thought about the visual relationship. Make sure that there is a focus on the vertical relationship when students are sharing.
- f. With the students, set up and solve the equation $h(x) = f(x) + k$ for the value of k and then write $h(x)$ in terms of f(x). Again, make sure that they understand that you can substitute x for $f(x)$ and $x - 5$ for $h(x)$.

For question #2: You Try

- Have the students work with a partner to answer all questions in #2.
- After approximately 10 minutes, have partners share their answers and justification for each part of problem #2 with the class.
- Have students think about and discuss with their partner how the graphs of the functions in both #1 and #2 are related to the k values. Call on individual students to share their thoughts and justifications to the question, "How does the value of k in $f(x) + k$ affect the graph of $f(x)$?"

For question #3: Compare the graphs of $f(x)$ and $f(x + k)$.

- a. With the students, graph $f(x) = 3x + 3$ and $g(x) = 3x + 6$.
- b and c. Have the students turn to a partner and determine the x-intercepts of $f(x)$ and $g(x)$. They should also discuss how the functions visually relate horizontally.
- d. With the students, set up and solve the equation $g(x) = f(x + k)$ for the value of k and then write $g(x)$ in terms of $f(x)$. Mention that the k is now inside of the parenthesis, so it may have a different effect on the graph. Make sure that they all understand that you are now substituting $(x + k)$ for the x in the equation $f(x) = 3x + 3$.
- e. With the students, graph $h(x)$. Tell the students to turn to their partner and discuss how the functions relate visually horizontally. Have students share, with the class, what their partner thought about the visual relationship. Make sure that there is a focus on the horizontal relationship when students are sharing.
- f. With the students, set up and solve the equation $h(x) = f(x + k)$ for the value of k and then write $h(x)$ in terms of $f(x)$. Again, make sure that they all understand that you are substituting $(x + k)$ for the x in the equation $f(x) = 3x + 3$.

Activity/Lesson Continued:

For question #4: You Try

- Have the students work with a partner to answer all questions in #4.
- After approximately 10 minutes, have partners share their answers and justification for each part of problem #4 with the class.
- Have students think about and discuss with their partner how the graphs of the functions in both #3 and #4 are related to the *k* values. Call on individual students to share their thoughts and justifications to the question, "How does the value of *k* in $f(x + k)$ affect the graph of $f(x)$?"

Check for Understanding

Before the next activity, you will want to formatively assess whether the students understand the two shifts. Ask these four questions.

1. Ask the students, "When the *k* is outside of the parenthesis, such as $f(x) + k$, how is the graph of $f(x)$ affected?"
   - Students should talk about *k* shifting the graph of $f(x)$ vertically (or up and down).
2. Now ask, "How is the graph of $f(x) + 8$ related to the graph of $f(x)$?"
   - Students should say, "The graph of $f(x)$ is shifted up 8 units."
3. Ask the students, "When the *k* is inside of the parenthesis, such as $f(x + k)$, how is the graph of $f(x)$ affected?"
   - Students talk about *k* shifting the graph of $f(x)$ horizontally (or left and right).
4. Now ask, "How is the graph of $f(x – 6)$ related to the graph of $f(x)$?"
   - You will want to hear, "The graph of $f(x)$ is shifted right 6 units."

Move on to the Shifting Linear Equations in Function Notation Sort

- Hand each pair of students an envelope/binder clip of sort pieces. Start by having students separate their sort pieces on their desk. They should have the two strips of graphs that they will line up on their desk and two piles of function pieces. One pile should be of the $f(x)$ pieces and the other pile should be of $g(x)$ pieces.
- Have students match the $f(x)$ pieces to the graphs first.
- Then they should use the $f(x)$ functions and the graphs to determine what $g(x)$ piece matches.
- Students who have finished matching all of the pieces should justify their answers. They can do so by substituting the $f(x)$ function into the $g(x)$ function, writing on the pieces of paper that have the $g(x)$ functions. They can then verify that the $g(x)$ function matches the graph.
- After all students are finished, or about 15 minutes has gone by, have students share with the class their matches and their justifications.

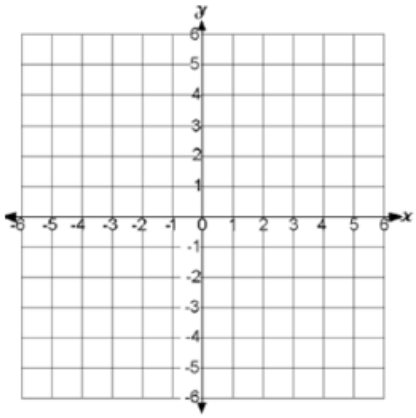
Extension/Closure: (on last page)

- These could be used to close a class, given to take home, or as review on another day.
- For #1, you want students to recognize that you could shift $f(x)$ down 5 units or to the right 5 units. That is why answer choice a and c are both correct.
- For #2, if students have trouble, you can create a $f(x)$ function and then have them create $g(x)$ and $h(x)$.

Warm-Up

Review: 8.F.A.3

Graph the linear equation $y = -2x + 5$



What is the slope of the line?

What is the y-intercept of the line?

Current: 8.F.A.3

a. Consider the parent function $y = x$.
How does $y = x + 4$ compare to $y = x$.

b. Rewrite $y = x + k$ for two other k values. Describe how each equation compares to the parent function $y = x$.

$y = x +$ _____

$y = x -$ _____

Review: HSF.BF.A.1C

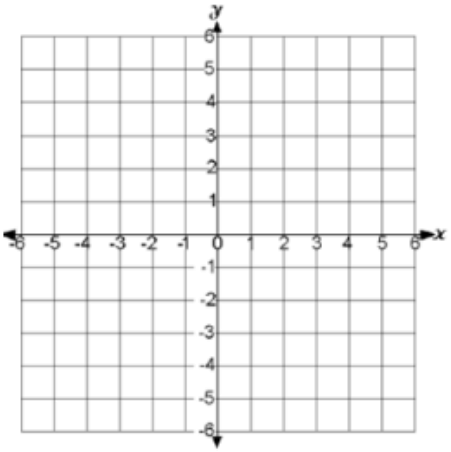
Match equivalent statements about $f(x) = 4x - 2$? Justify your selections.

1. $f(x + 2)$	a. $(4x - 2) - 2$
2. $f(x) - 2$	b. $4(x - 2) - 2$
3. $f(x) + 2$	c. $4x$
4. $f(x - 2)$	d. $4x + 6$

Review: HS.IF.C7.A

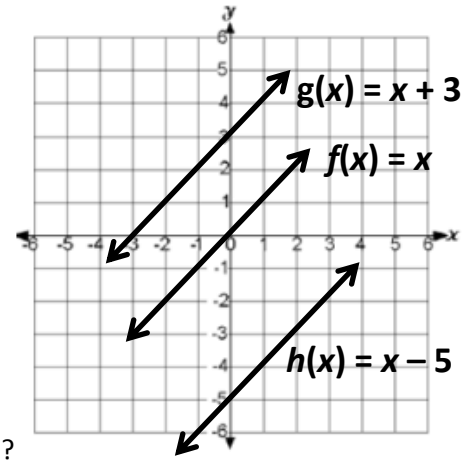
Fill out the table for $f(x) = 3x - 2$ for 5 values of x and use the ordered pairs to graph the linear function.

x	$f(x)$
-2	-8
-1	-5
0	-2
1	1
2	4



Name: ___Key/Soultions_____

Shifting Linear Equations in Function Notation



1. Compare the graphs of *f*(*x*) and *f*(*x*) + *k*.

a. Graph functions $f(x) = x$ and $(x) = x + 3$.

b. What are the similarities and differences of $f(x)$ and $g(x)$?

Same slope, different *y*-intercept

c. Visually, how does $g(x)$ relate to $f(x)$?

***g*(*x*) is equivalent to moving *f* (*x*) up 3 units**

d. If $g(x) = f(x) + k$, find the value of k and write $g(x)$ in terms of $f(x)$.

$g(x) = f(x) + k$

$x + 3 = x + k$ ***(substitute x + 3 for g(x) and x for f(x))***

$k = 3$

$\therefore g(x) = f(x) + 3$

e. Graph the function $h(x) = x - 5$. Visually, how does $h(x)$ relate to $f(x)$?

***h*(*x*) is equivalent to moving *f*(*x*) down 5 units**

f. If $h(x) = f(x) + k$, find the value of k and write $h(x)$ in terms of $f(x)$.

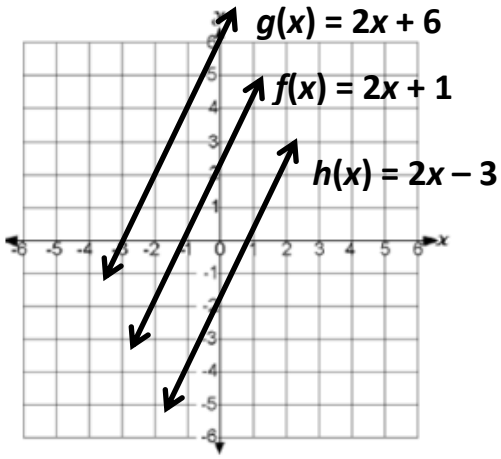
$h(x) = f(x) + k$

$x - 5 = x + k$ ***(substitute x – 5 for h(x) and x for f(x))***

$k = -5$

$\therefore h(x) = f(x) - 5$

2. You Try



a. Graph the functions $f(x) = 2x + 1$ and $g(x) = 2x + 6$.

b. What are the similarities and differences of $f(x)$ and $g(x)$?

Same slope, different *y* intercept

c. Visually, how is $f(x)$ and $g(x)$ related?

***g*(*x*) is equivalent to *f*(*x*) moved up 5 units**

d. If $g(x) = f(x) + k$, find the value of k and write $g(x)$ in terms of $f(x)$.

$g(x) = f(x) + k$

$2x + 6 = 2x + 1 + k$ ***(substitute 2x + 6 for g(x) and 2x + 1 for f(x))***

$6 = 1 + k$

$k = 5$

$\therefore g(x) = f(x) + 5$

e. Let $h(x) = 2x - 3$. Visually, how does $h(x)$ relate to $f(x)$?

***h*(*x*) is the same as *f*(*x*) moved down 4 units**

f. If $h(x) = f(x) + k$, find the value of k and write $h(x)$ in terms of $f(x)$.

$h(x) = f(x) + k$

$2x - 3 = 2x + 1 + k$ ***(substitute 2x – 3 for h(x) and 2x + 1 for f(x))***

$-3 = 1 + k$

$k = -4$

$h(x) = f(x) - 4$

How does the value of *k* in *f*(*x*) + *k* affect the graph of *f*(*x*)? (Considering the functions in #1 and #2)
The *k* value in *f*(*x*) + *k* will shift the graph of *f*(*x*) vertically *k* units. Positive *k* shifts *f*(*x*) up *k* units. Negative *k* will shift *f*(*x*) down *k* units.

3. Compare the graphs of $f(x)$ and $f(x + k)$.

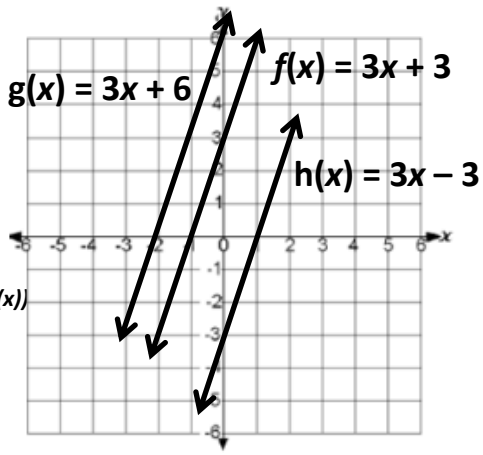
a. Graph the functions $f(x) = 3x + 3$ and $g(x) = 3x + 6$.

b. What are the x-intercepts of $f(x)$ and $g(x)$?

$f(x)$ is at $x = -1$ and $g(x)$ is at $x = -2$

c. Visually, how does $g(x)$ relate to $f(x)$ horizontally?

$g(x)$ is equivalent to moving $f(x)$ left 1 unit



d. If $g(x) = f(x + k)$, find the value of k and write $g(x)$ in terms of $f(x)$.

$g(x) = f(x + k)$ *(f(x) = 3x + 3))*

$3x + 6 = 3(x + k) + 3$ *(substitute 3x + 6 for h(x) and replace x with x + k in f(x))*

$3x + 6 = 3x + 3k + 3$

$6 = 3k + 3$

$3 = 3k$

$k = 1$.

e. Graph the function $h(x) = 3x - 3$. Visually, how does $h(x)$ relate $f(x)$ to horizontally?

$h(x)$ is equivalent to moving $f(x)$ right 2 units

f. If $h(x) = f(x + k)$, find the value of k and write $g(x)$ in terms of $f(x)$.

$h(x) = f(x + k)$ *(f(x) = 3x + 3))*

$3x - 3 = 3(x + k) + 3$ *(substitute 3x – 3 for h(x) and replace x with x + k in f(x))*

$3x - 3 = 3x + 3k + 3$

$-3 = 3k + 3$

$-6 = 3k$

$k = -2$

4. You Try

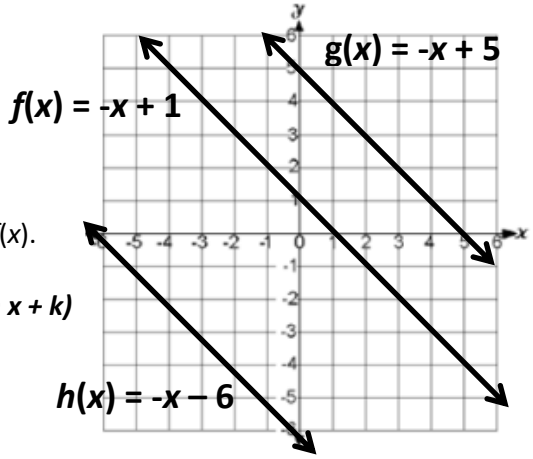
a. Graph the functions $f(x) = -x + 1$ and $g(x) = -x + 5$.

b. What are the x-intercepts of $f(x)$ and $g(x)$?

$f(x)$ is at $x = 1$ and $g(x)$ is at $x = 5$

c. Visually, how does $g(x)$ relate to $f(x)$ horizontally?

$g(x)$ is equivalent to moving $f(x)$ right 4 units



d. If $g(x) = f(x + k)$, find the value of k and write $g(x)$ in terms of $f(x)$.

$g(x) = f(x + k)$ *(f(x) = -x + 1)*

$-x + 5 = -(x + k) + 1$ *(g(x) = -x + 5 and replace x in f(x) with x + k)*

$-x + 5 = -x - k + 1$

$5 = -k + 1$

$4 = -k$

$k = -4$ $\therefore g(x) = f(x) - 4$

e. Graph the function $h(x) = -x - 6$. Visually, how does $h(x)$ relate to $f(x)$ horizontally?

$h(x)$ is equivalent to moving $f(x)$ left 7 units

f. If $h(x) = f(x + k)$, find the value of k and write $h(x)$ in terms of $f(x)$.

$h(x) = f(x + k)$ *(f(x) = -x + 1)*

$-x - 6 = -(x + k) + 1$ *(h(x) = -x – 6 and replace x in f(x) with x + k)*

$-x - 6 = -x - k + 1$

$-6 = -k + 1$

$-7 = -k$

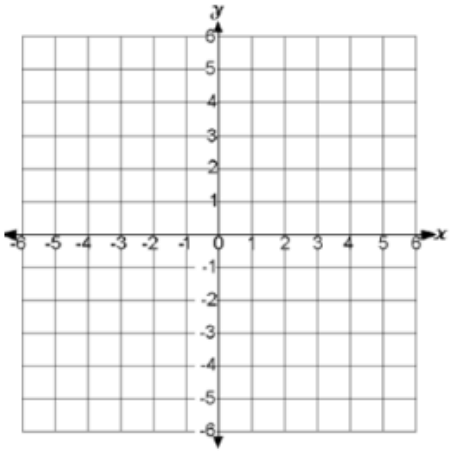
$k = 7$ $\therefore g(x) = f(x + 7)$

How does the value of k in $f(x + k)$ affect the graph of $f(x)$? (Considering the functions in #3 and #4)

The k value in $f(x + k)$ will shift the graph of $f(x)$ horizontally k units. Positive k shifts $f(x)$ left k units. Negative k will shift $f(x)$ right k units.

Name: _____

Shifting Linear Equations in Function Notation
Note Taking Guide



1. Compare the graphs of *f*(*x*) and *f*(*x*) + *k*.

a. Graph functions $f(x) = x$ and $g(x) = x + 3$.

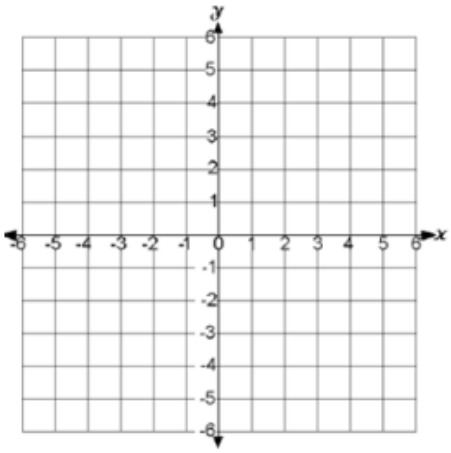
b. What are the similarities and differences of $f(x)$ and $g(x)$?

c. Visually, how does $g(x)$ relate to $f(x)$?

d. If $g(x) = f(x) + k$, find the value of k and write $g(x)$ in terms of $f(x)$.

e. Graph the function $h(x) = x - 5$. Visually, how does $h(x)$ relate to $f(x)$?

f. If $h(x) = f(x) + k$, find the value of k and write $h(x)$ in terms of $f(x)$.



2. You Try

a. Graph the function $f(x) = 2x + 1$ and $g(x) = 2x + 6$.

b. What are the similarities and differences of $f(x)$ and $g(x)$?

c. Visually, how is $f(x)$ and $g(x)$ related?

d. If $g(x) = f(x) + k$, find the value of k and write $g(x)$ in terms of $f(x)$.

e. Let $h(x) = 2x - 3$. Visually, how does $h(x)$ relate to $f(x)$?

f. If $h(x) = f(x) + k$, find the value of k and write $h(x)$ in terms of $f(x)$.

How does the value of *k* in *f*(*x*) + *k* affect the graph of *f*(*x*)? (Considering the functions in #1 and #2)

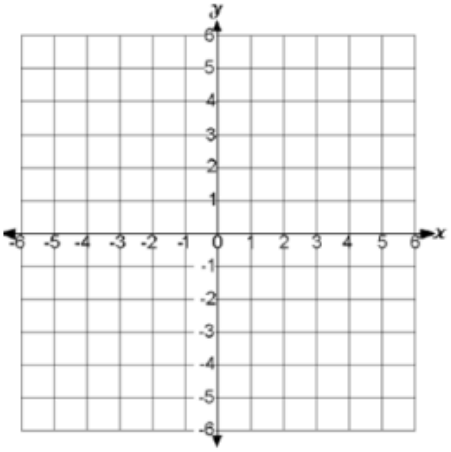
3. Compare the graphs of *f*(*x*) and *f*(*x* + *k*).

a. Graph the functions $f(x) = 3x + 3$ and $g(x) = 3x + 6$.

b. What are the *x*-intercepts of $f(x)$ and $g(x)$?

c. Visually, how does $g(x)$ relate to $f(x)$ horizontally?

d. If $g(x) = f(x + k)$, find the value of *k* and write $g(x)$ in terms of $f(x)$.



e. Graph the function $h(x) = 3x - 3$. Visually, how does $h(x)$ relate to $f(x)$ horizontally?

f. If $h(x) = f(x + k)$, find the value of *k* and write $h(x)$ in terms of $f(x)$

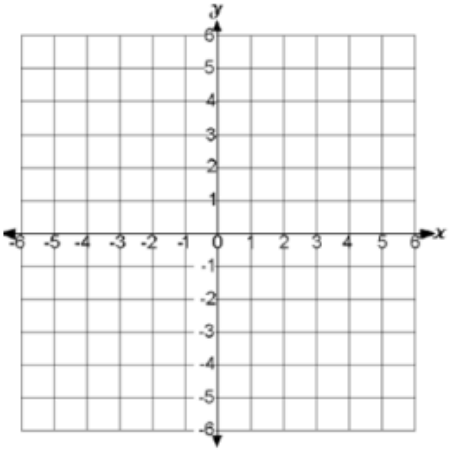
4. You Try

a. Graph the functions $f(x) = -x + 1$ and $g(x) = -x + 5$.

b. What are the *x*-intercepts of $f(x)$ and $g(x)$?

c. Visually, how does $g(x)$ relate to $f(x)$ horizontally?

d. If $g(x) = f(x + k)$, what is the value of *k*? Justify your answer.

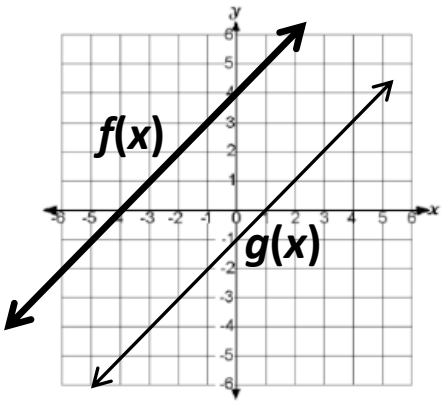
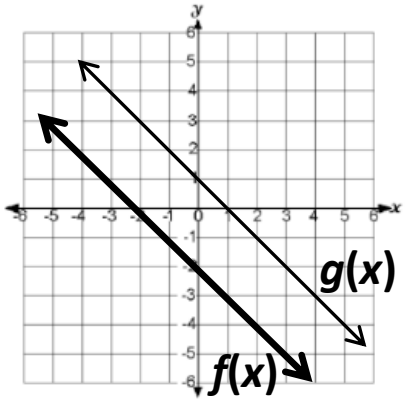
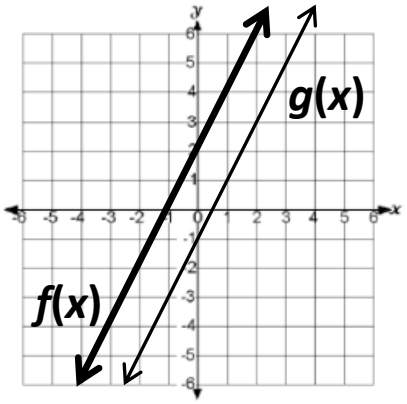


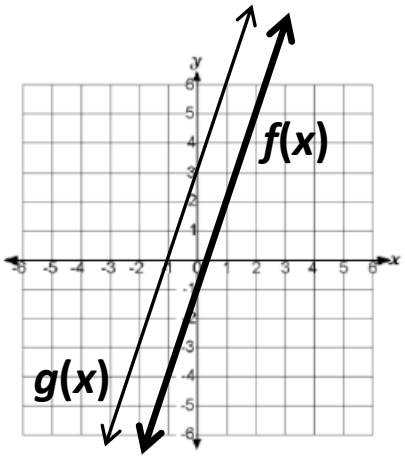
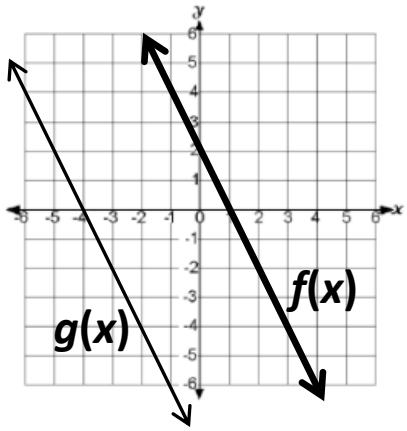
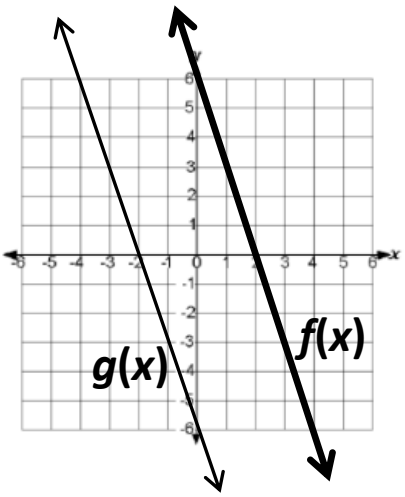
e. Graph the function $h(x) = -x - 6$. Visually, how does $h(x)$ relate to $f(x)$ horizontally?

f. If $h(x) = f(x + k)$, what is the value of *k?* Justify your answer.

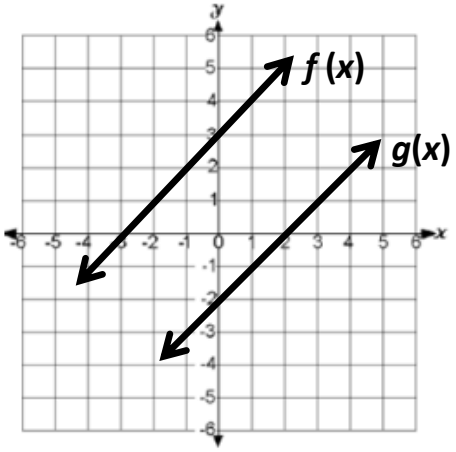
How does the value of *k* in *f*(*x* + *k*) affect the graph of *f*(*x*)? (Considering the functions in #3 and #4)

Shifting Linear Equations in Function Notation Sort

	$f(x) = x + 4$	$g(x) = f(x) - 5$
	$f(x) = -x - 2$	$g(x) = f(x-3)$
	$f(x) = 2x + 2$	$g(x) = f(x) - 3$

	$f(x) = 3x - 1$	$g(x) = f(x) + 4$
	$f(x) = -2x + 2$	$g(x) = f(x+5)$
	$f(x) = -3x + 6$	$g(x) = f(x+4)$

Extension/Closure:

1. Which equation(s) correctly relates $f(x)$ to $g(x)$? Justify your answer(s).  a. $g(x) = f(x - 5)$ b. $g(x) = f(x + 5)$ c. $g(x) = f(x) - 5$ d. $g(x) = f(x) + 5$	2. Apply what you have learned to create $f(x)$, $g(x)$, and $h(x)$ to meet the following conditions. Show all of your work. a. Create an equation in the form $f(x) = mx + b$. *b.* Write the equation of $g(x)$, such that $g(x)$ is equivalent to shifting $f(x)$ down 3 units. c. Write the equation of $h(x)$, such that $h(x)$ is equivalent to shifting $f(x)$ right 2 units.

Key/Solutions

1. a and c **a. $f(x - 5)$ would shift $f(x)$ to the right 5 units** **c. $f(x) - 5$ would shift $f(x)$ down 5 units**	**2. Answers may vary** **Possible answer:** **a. $f(x) = 2x + 1$** **b. $g(x) = f(x) - 3$** **$g(x) = 2x + 1 - 3$** **$g(x) = 2x - 2$** **c. $h(x) = f(x - 2)$** **$h(x) = 2(x - 2) + 1$** **$h(x) = 2x - 4 + 1$** **$h(x) = 2x - 3$**